b.) 10^{35} , 10^{26}

Solution: $10^{35}/10^{26} = 10^{35-26} = 10^{9}$

So 10^{35} is 10^{9} times larger than 10^{26}.

c.) 2×10^{-9} , 2×10^{-6}

Solution: $2\times10^{-9}/2\times10^{-6} = \frac{\cancel{2}\times10^{-9}}{\cancel{2}\times10^{-6}}$

$= 10^{-9}\cdot10^{6} = 10^{-3}$

So 2×10^{-9} is 10^{3} times smaller than 2×10^{-6}

§ 3C Dealing with Uncertainty

A review of significant digits:

Def: Digits in a number that represent actual measurements are termed significant.

Determining significance:

1.) Zeros between nonzero digits are always significant.
2.) Nonzero digits are always significant
3.) Zeros following a nonzero digit and residing to the right of the decimal point are always significant
4.) Zeros to the left of the first nonzero digit are Never significant.
5.) Zeros to the right of the last nonzero digit but before the decimal point are not significant unless stated otherwise

Ex: Count the number of significant digits:

a.) 23 2 sig fig
b.) 2.01 3 sig fig
c.) 1,000 1 sig fig
d.) 0.0095 2 sig fig
e.) 3×10^{-2} 1 sig fig
f.) 0915.01 5 sig fig
g.) 0.100 3 sig fig

Understanding Errors in Measurement

There are two types of errors:

Random Errors: Errors caused by random, unpredictable events during the measurement process

Systemic Errors: Errors caused by a problem in the measurement system that influence the results. Ex: Failing to "zero" a scale prior to taking a weight measurement.

Ex: Think about some of the possible random and systemic errors that could occur in the following situations:

a.) The average income of 25 people found by checking their tax returns

Random error: Misreading numbers on the forms; failure to calculate the average correctly.
Systemic error: Error from people over or under reporting their incomes.

b) Times in a swimming meet:

Random error: Error from misreading the watch.
Systemic error: Error from the watch not being calibrated correctly.

Considering the size of Errors:

The following formulas will look VERY FAMILIAR:

$$\text{Absolute Error} = \text{Measured value} - \text{true value}$$

$$\text{relative Error} = \frac{\text{Absolute Error}}{\text{True value}}$$

As before, we can change relative error to a percent error by multiplying by 100%

Ex: You purchase a 50 lb bag of dog food. The actual weight is 51 lbs. What are the Absolute and Relative Error?

Solution:

Absolute: $50\,lb - 51\,lb = -1\,lb$

Relative: $-\dfrac{1\,lb}{51\,lb} \approx -.02 = -2\%$

So the measured value is 2% less than the actual value.

Accuracy vs. Precision in Measurement

Precision describes the level of detail in a measurement.

Ex: Suppose you have a digital thermometer and an analog thermometer. Suppose the analog thermometer measures to the nearest degree F. Suppose the digital thermometer measures to the nearest tenth of a degree F. Then for instance:

ANALOG: $32°F$

DIGITAL: $32.23°F$

The digital thermometer is more precise.

Accuracy describes how closely a measurement compares with the "actual" value.

Ex: Suppose the distance from A to B is 3.7 cm

Using a standard ruler you measure a length of

Standard ruler: 3.6 cm

With an advanced laser measurement device you get a measure of:

Laser device: 3.492 cm

Since 3.6 is closer to 3.7 than 3.492, the ruler is more accurate. Notice the laser device is more precise since it measures to the nearest thousandth of a cm while the ruler measures only to the nearest tenth of a centimeter.

Ex: Your true height is 70.50 inches. A tape measure accurate to the nearest $\frac{1}{8}$ inch gives your height as $70\frac{3}{8}$ inches. A laser device gives your height as being 70.90 inches, accurate to the nearest 0.05 inches. Which is more accurate? Precise?

Solution:

Notice $\frac{5}{100} = .05$ is smaller than $\frac{1}{8}$

Thus the laser is more precise.

For accuracy we have $\frac{3}{8} = .375$.

So, the following are our measurements:

$TM = 70\frac{3}{8} in = 70.375 \, in$

$LD = 70.9 \, in$

Since $70.90 \, in - 70.50 \, in = 0.4 \, in$

$70.375 \, in - 70.50 = -0.125 \, in$

Since .4 in is larger than .125 in (ignore the minus sign, it only tells us the tape measure gave a height below the tree height), we conclude the Tape measure is more accurate.

The Rounding rules for Combining measured numbers.

Rounding rule for + + - : Round the answer to the same precision as the least precise number in the problem.

Ex: 23.57 °F + 15.1 °F

Solution:

$$\begin{array}{r} 23.57\,°F \\ +\ 15.1\,°F \\ \hline 38.67\,°F \end{array}$$

Since 15.1 °F is precise only to the nearest tenth of a °F, our answer should be the same. IE:

$\boxed{38.7\ °F}$

Ex: 105 km - 3.15 km

Solution

$$\begin{array}{r} \overset{4\,9\,10}{105.00}\ km \\ -\ \ 3.15\ km \\ \hline 101.85\ km \end{array}$$

Since 105 km is accurate only to the nearest km, our answer is only accurate to the nearest km. IE:

$\boxed{102\ km}$

Rounding rule for $\times$ and $\div$: Round your answer to the same number of significant digits as the answer with the fewest significant digits.

Ex: $15\,km\overline{)13.5\ km}$

Solution:

$$\frac{13.5\ km}{15\ km} = .9$$

Since 15 km has 2 sig. digits and 13.5 km has 3, our answer needs two sig digits.

$\boxed{.90}$

Ex: .5 ft x 152 ft

Solution:

$$\begin{array}{r} 152\ ft \\ \times\ .5\ ft \\ \hline 76.0\ ft^2 \end{array}$$

Since .5 ft has only 1 sig digit, our answer can have only one sig digit. so; round $76.0\ ft^2$:

$\boxed{80\ ft^2}$

Note the zero here is not significant.

NOTE: YOU SHOULD APPLY THE ROUNDING ONLY AFTER COMPLETING THE $\div, \times, +,$ or $-$. FAILING TO DO SO COULD RESULT IN ROUNDING ERROR.